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Analytical Solution for Hoop Tension and Bending Moment Coefficient for Long Cylinders by Beam on Elastic Foundation

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Abstract-The respective countries design codes provide the coefficients for the bending moment, hoop tension and shear force by the dimensionless parameter H^2/Dt. The IS: 3370 - 1967 (Part IV) provide the design tables and the respective coefficients for the range of $H^2/Dt = 0.4$ to 16. This dimensionless parameter is a predominant parameter in design of cylindrical shells since the coefficients are dependent on it. When the H^2/Dt is more than 16 the coefficients are not provided in IS: 3370 and extrapolating the coefficients may lead to erroneous results. This paper calculates the bending moment and hoop tension coefficients for H2/Dt from 0.4 to 100 by utilizing the general solutions of beam on elastic foundation provided by Timoshenko (1940) and M. Hetènyi (1979).

Index Terms–Beam on elastic foundation, bending moment coefficient, Circular cylindrical shells, hoop tension coefficient, IS: 3370 – 1967 (Part IV).

1. INTRODUCTION

The shells are used to contain or resist the liquid or solid materials from inside or from outside. Due to this the hoop tension, bending moment and shear force were developed in the shell, to resist these structural actions, code of the respective countries provide design coefficients corresponding to H^2/Dt with respect to the height of the shell or tank as the behavioural study, analysis and design of the shell is a complicated procedure.

Cylindrical wall of the shells are subjected to the radial pressure due to the stored materials or due to supporting the soil which is present on external of the shell. The pressure is assumed to have an intensity same in one level and varies along vertical direction of the shell. As the radial displacement is accompanied by the hoop forces, the small element of the cylindrical shell behaves like a beam on elastic foundation which gives reaction forces proportional to the deflection of the wall. The current paper is restricted to the cylindrical shell resting on the ground.

The need of this work is found because as the dimensionless parameter H^2/Dt ratio is more than 16 the code [4] has not recommended any method for the analysis. And the extrapolation from available coefficients will lead to the erroneous results. And as the designers seem to face the difficulty in the analysis, the calculated coefficients serves as the useful tool for the practicing design engineers and research workers who work in the field of the cylindrical shells or tanks or wells or pressure vessels.

Edmund S. Melerski (1991) [5] carried out simple elastic analysis of axisymmetric cylindrical storage tanks, in his work he has analyzed the axisymmetrical loaded cylindrical shells by the force method and generated the computer programme.

Anand Daftardar, Shirish Vichare, Jigisha Vashi (2017) [3], carried out Analytical Solution for Hoop Tension in Liquid Storage Cylindrical Tanks. In this work they have calculated the hoop tension coefficient by the classical bending theory and the boundary conditions used are bottom pinned and top free and the tank is resting on ground. They have calculated the hoop coefficients for dimensionless parameter H^2/Dt from 0 to 100.

Timoshenko (1940) [1], in his strength of materials part 2 has provided the general solution for beam on elastic foundation.

M. Hetènyi (1979) [2], in his book beam on elastic foundation has provided the general solutions.

The current work calculates the bending moment and hoop coefficients from the classical theory presented by the Timoshenko (1940) [1] and M. Hetènyi (1979) [2] for dimensionless parameter $H^2/Dt = 0.4$ to 16. After 16 the ratio was selected in the intervals of five that is from 20 to 100 in the interval of the five the bending moment and hoop tension coefficients were calculated.

2. BEAM ON ELASTIC FOUNDATION

When flexural rigidity of the cylindrical tank is taken into account, a solution can be used that is based on some form of a beam on an elastic foundation.

The boundary condition is bottom fix and top free. The shell is subjected to the triangular hydrostatic pressure.

Consider a strip of a cylindrical shell subjected to the load perpendicular to its axis in the principal plane of the symmetrical cross section as shown in fig 1. Because of this the wall of shell will deflect, producing continuously distributed reaction forces in the medium. Regarding these reaction forces we make the fundamental assumption that their intensity p at any point is proportional to the deflection of the beam y at that point,

$$p= ky$$

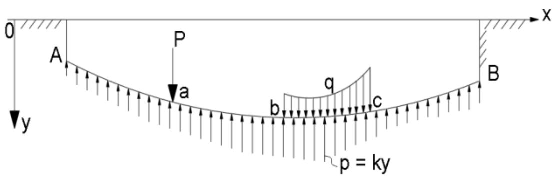

Figure 1

Let us take a small element enclosed between two vertical cross sections a distancedx apart on the beam under consideration. Assume that this element was taken from a portion where the shell was acted upon by a distributed loading q. the forces exerted on such an element was shown in fig 2.

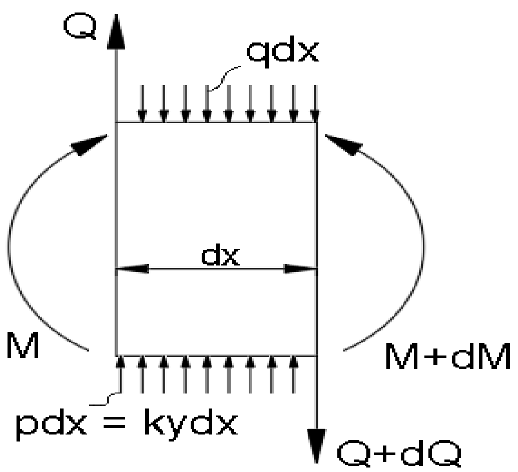

Figure 2

The reaction force Q is considered as positive, as is corresponding to bending moment, M, which is clockwise moment acting from left on the element (the moment of a positive Q). These positive directions for Q and M will be kept in all later derivations.

Considering the equilibrium of the element in fig 8, we find that the summation of the vertical forces gives,

$$Q - (Q + dQ) + ky\, dx - q\, dx = 0$$

Hence,

$$dQ/dx = ky - q$$

Making use of the relation $Q = dM/dx$, we can write,

$$dQ/dx = d^2M/dx^2 = ky - q \qquad (1)$$

Using now the known differential equation of a beam in bending, $EI\,(d^2y/dx^2) = -M$,

And differentiating it twice, we obtain

$$EI\,(d^4y/dx^4) = -\,(d^2M/dx^2)$$

Hence by using equation(1)We find

$$EI\,(d^4y/dx^4) = -ky + q$$

This is the differential equation for the deflection curve of a beam supported on elasticfoundation. Along the unloaded potion of beam, the equation takes form

$$EI\,(d^4y/dx^4) = -ky \qquad (2)$$

The general solution of (2) takes the form

$$Y = A_1e^{mx} + A_2\, e^{m2x} + A_3\, e^{m3x} + A_4\, e^{m4x} (3)$$

We can write (3) in more convenient form

$$y = e^{\mathbf{b}x}\,(C_1\cos\mathbf{b}x + C_2\, \sin \mathbf{b}x) + e^{-bx}\,(C_3\cos\mathbf{b}x + C_4\, \sin \mathbf{b}x) + f\,(x) \qquad (4)$$

where,

$\beta = \sqrt[4]{\frac{k}{4EI}}$ is a characteristic factor

C_n is integration constant which are obtained after applying boundary condition.

Where, EIis flexural rigidity,

Let us say EI = I

$$\mathbf{I} = \frac{\mathbf{Et^3}}{\mathbf{12(1-\vartheta^2)}}$$

3. ANALYTICAL MODELLING

Cylindrical shell cross section is shown in figure 3. The bottom is fixed and top is free. Shell is subjected to hydrostatic pressure.

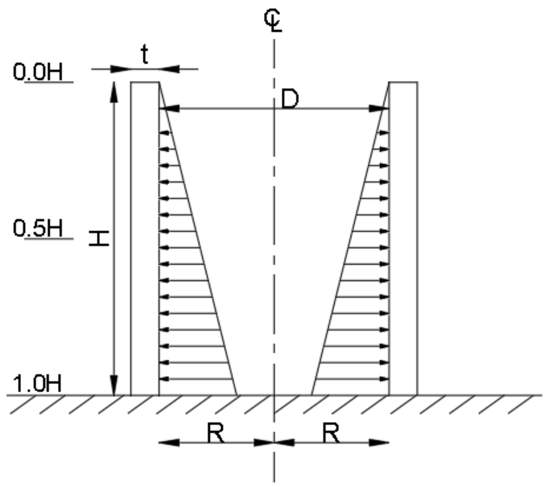

Figure 3

Due to this pressure the shell is subjected to hoop tension, bending and outward deflection.

Using equation (4), in which f(x) is

$$\gamma\,(H - x)$$

The boundary conditions are,

At x = H, M = 0 therefore $EI\frac{d^2y}{dx^2} = 0$

At x = H, Q = 0 therefore $EI\frac{d^3y}{dx^3} = 0$

At x = 0, θ = 0 therefore $EI\frac{dy}{dx} = 0$

At x = 0, y = 0 therefore EIy = 0

Deflection is,

$$y = -\frac{\gamma R^2}{Et}\left[1 - \frac{x}{H} - \theta(\beta x) - \left(1 - \frac{1}{\beta H}\right)\zeta(\beta x)\right] \tag{5}$$

$\theta(\beta x)$ And $\zeta(\beta x)$ are calculated with the help of table 1 page no 5 of reference [1].

Hoop tension is,

$$T = \frac{Ety}{R} \tag{6}$$

And as per reference [4],

$$T = \text{Coefficient x } \gamma \text{ x H x R} \tag{7}$$

Bending moment is,

$$M = I\frac{d^2y}{dx^2} \tag{8}$$

And as per reference [4],

$$M = \text{Coefficient x } \gamma \text{ x } H^3 \tag{9}$$

The hoop and bending moment coefficients were calculated from equation (7) and (9) respectively after substituting the hoop and bending moment value obtained from equation (6) and (8).

4. VALIDATION AND EXTENSION OF COEFFICIENTS

The hoop tension and bending moment coefficients for fixed base and free top are give in table 9 and 10 of reference [4]. The coefficients provided by reference [4] are for H^2/Dt ranging from 0.4 to 16. And these coefficients were used for the validation of coefficients which were obtained by the beam on elastic foundation.

The data input taken as the following,

Density of water γ = 9.81 kN/m^3

Poisson ratio = 0.15

Modulus of elasticity = 900666.42 kN/m^2

The calculated coefficients for the range of H^2/Dt 0.4 to 16 are matching with IS: 3370 (Part IV). Hence the concept beam on elastic foundation is revalidated and the range of H^2/Dt is extended from 20 to 100 with an interval of five. The calculated coefficients are in appendix A, and these coefficients are named as Imran and Gupta Coefficients.

5. DISCUSSION AND CONCLUSION

1. The results obtained from the IS code for cylindrical shells having $H^2/Dt < 3$ for bottom fixed support and top free, shows that upper part of the wall is having max hoop tension in 0.4H from top and lower part is having max bending in lower region of 0.7H. The bottom part of the shell will bend and the top part of the shell will bulge out due to the hydrostatic pressure from inside. In these shells the bending is predominant from design point of view.
2. The results obtained from the IS code for cylindrical shells having range of H^2/Dt 3 to 16 for bottom fixed support and top free, shows the wall is having more hoop tension in 0.3H to 0.8H from top and lower part is having max bending in lower region of 0.7H. In these kind of shells the design has to be done for both hoop and bending.
3. As the range of the range of H^2/Dt is more than 16 the bending moment will be shifted from 0.7H to 1.0H. And the hoop tension will lie in the region of 0.5H to 0.8H.
4. The extension of the beam on elastic for H^2/Dt = 20 to 100 with an interval of the 5 was carried out and it is seen that as the ratio H^2/Dt increases, the hoop is becoming most prime criteria for the design of the cylindrical shells. And the bending is only in bottom region of 0.9H to 1.0H. The bending is dying out after 0.7H and hoop is dominant from 0.8H to 0.3H.
5. With the use of beam on elastic foundation the bending moment and hoop tension coefficients are calculated and the obtained values are matching with the reference [4]. Hence the obtained coefficients given in the table A.1 and A.2 of appendix A. The obtained coefficients for the range of H^2/Dt 20 to 100 may be found useful for the practicing engineers and research workers.

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Appendix A

Table A.1: Bending Moment Coefficients for Circular Ring Wall Fixed Base and Free Top Subjected to Triangular Load

H^2/Dt	Bending Moment Coefficients at Points										
	1.0H Bottom	0.9H	0.8H	0.7H	0.6H	0.5H Mid	0.4H	0.3H	0.2H	0.1H	0.0H Top
0.4	-0.0601	-0.0028	-0.0036	-0.0028	0.002	0.0099	0.0175	0.027	0.0376	0.047	0.0569
0.8	-0.0722	-0.0044	-0.0097	-0.0107	-0.002	-0.011	0.0009	0.007	0.0134	0.0197	0.0255
1.2	-0.0615	-0.0042	-0.0092	-0.0103	-0.0087	-0.0053	-0.0011	0.0034	0.008	0.0121	0.0157
1.6	-0.0523	-0.004	-0.0087	-0.0095	-0.0079	-0.0046	-0.0008	0.003	0.0065	0.0096	0.0121
2	-0.0451	-0.0043	-0.0091	-0.0095	-0.0075	-0.0042	-0.0004	0.0033	0.0065	0.009	0.011
3	-0.0335	-0.0035	-0.0068	-0.0067	-0.0047	-0.0019	0.0008	0.003	0.0047	0.0059	0.0065
4	-0.0261	-0.0031	-0.0057	-0.0051	-0.003	-0.0007	0.0013	0.0028	0.0037	0.0042	0.0044
5	-0.0221	-0.0028	-0.005	-0.0041	-0.0021	-0.0001	0.0015	0.0025	0.0031	0.0033	0.0033
6	-0.0187	-0.0026	-0.0044	-0.0034	-0.0015	0.0003	0.0015	0.0022	0.0025	0.0026	0.0025
8	-0.0148	-0.0019	-0.003	-0.002	-0.0006	0.0005	0.0012	0.0015	0.0015	0.0015	0.0034
10	-0.012	-0.0023	0.0033	-0.0019	-0.0002	0.0009	0.0014	0.0015	0.0014	0.0014	0.0013
12	-0.0103	-0.0019	-0.0025	-0.0012	0	0.0008	0.001	0.001	0.001	0.0006	0.0012
14	-0.0089	-0.0015	-0.0019	-0.0008	0.0002	0.0006	0.0008	0.0007	0.0007	0.0006	0.0006
16	-0.0079	-0.0014	-0.0016	-0.0006	0.0002	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
20	-0.0064	-0.0014	-0.0015	-0.0003	0.0004	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004
25	-0.0052	-0.0012	-0.001	-0.0001	0.0003	0.0004	0.0004	0.0003	0.0003	0.0003	0.0003
30	-0.0044	-0.0012	-0.0009	0	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003
35	-0.0038	-0.001	-0.0007	0.0001	0.0003	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
40	-0.0034	-0.001	-0.0005	0.0001	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
45	-0.003	-0.0009	-0.0004	0.0001	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
50	-0.0027	-0.0009	-0.0004	0.0001	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001
55	-0.0025	-0.0008	-0.0003	0.0001	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
60	-0.0023	-0.0008	-0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
65	-0.0021	-0.0007	-0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
70	-0.002	-0.0007	-0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
75	-0.0018	-0.0007	-0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
80	-0.0017	-0.0006	-0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
85	-0.0016	-0.0006	-0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
90	-0.0015	-0.0006	-0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
95	-0.0015	-0.0006	0	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
100	-0.0014	-0.0006	0	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001

Table A.2: Hoop Tension Coefficients for Circular Ring Wall Fixed Base and Free Top Subjected to Triangular Load

H^2/Dt	Hoop Tension Coefficients at Points										
	1.0H Bottom	**0.9H**	**0.8H**	**0.7H**	**0.6H**	**0.5H Mid**	**0.4H**	**0.3H**	**0.2H**	**0.1H**	**0.0H Top**
0.4	0	-0.002	-0.005	-0.007	-0.006	0.002	0.016	0.038	0.068	0.106	0.152
0.8	0	-0.009	-0.029	-0.051	-0.055	-0.077	-0.076	-0.061	-0.034	0.006	0.059
1.2	0	0.017	0.054	0.096	0.131	0.153	0.157	0.143	0.111	0.062	-0.001
1.6	0	0.025	0.079	0.138	0.187	0.216	0.221	0.202	0.162	0.102	0.027
2	0	0.034	0.104	0.178	0.237	0.269	0.272	0.246	0.196	0.126	0.041
3	0	0.054	0.161	0.266	0.338	0.368	0.356	0.309	0.236	0.144	0.042
4	0	0.076	0.218	0.345	0.42	0.438	0.406	0.337	0.244	0.139	0.029
5	0	0.093	0.261	0.4	0.472	0.477	0.427	0.343	0.241	0.13	0.019
6	0	0.114	0.307	0.455	0.519	0.506	0.44	0.343	0.232	0.12	0.009
8	0	0.147	0.379	0.532	0.575	0.534	0.443	0.333	0.218	0.107	-0.151
10	0	0.182	0.444	0.593	0.611	0.543	0.436	0.32	0.208	0.101	-0.002
12	0	0.212	0.496	0.635	0.629	0.543	0.428	0.313	0.202	0.137	-0.002
14	0	0.242	0.544	0.667	0.639	0.538	0.42	0.306	0.199	0.098	-0.001
16	0	0.269	0.579	0.691	0.643	0.535	0.413	0.302	0.198	0.098	-0.001
20	0	0.322	0.65	0.721	0.641	0.52	0.404	0.299	0.198	0.099	0
25	0	0.38	0.709	0.738	0.632	0.509	0.399	0.298	0.199	0.1	0
30	0	0.431	0.751	0.743	0.623	0.503	0.398	0.299	0.2	0.1	0
35	0	0.477	0.782	0.742	0.615	0.5	0.398	0.299	0.2	0.1	0
40	0	0.515	0.802	0.738	0.609	0.499	0.399	0.3	0.2	0.1	0
45	0	0.555	0.819	0.732	0.604	0.498	0.399	0.3	0.2	0.1	0
50	0	0.589	0.829	0.727	0.602	0.498	0.4	0.3	0.2	0.1	0
55	0	0.621	0.836	0.721	0.6	0.499	0.4	0.3	0.2	0.1	0
60	0	0.648	0.84	0.717	0.599	0.499	0.4	0.3	0.2	0.1	0
65	0	0.673	0.843	0.713	0.598	0.499	0.4	0.3	0.2	0.1	0
70	0	0.696	0.843	0.71	0.598	0.5	0.4	0.3	0.2	0.1	0
75	0	0.718	0.843	0.707	0.598	0.5	0.4	0.3	0.2	0.1	0
80	0	0.737	0.842	0.705	0.598	0.5	0.4	0.3	0.2	0.1	0
85	0	0.755	0.84	0.703	0.599	0.5	0.4	0.3	0.2	0.1	0
90	0	0.772	0.838	0.701	0.599	0.5	0.4	0.3	0.2	0.1	0
95	0	0.787	0.836	0.7	0.599	0.5	0.4	0.3	0.2	0.1	0
100	0	0.801	0.833	0.7	0.599	0.5	0.4	0.3	0.2	0.1	0